ANALYTICAL SOLUTION BASED ON HYPERGEOMETRIC FUNCTION FOR SPECTRAL FATIGUE LIFE PREDICTION UNDER BROAD-BAND RANDOM LOADING

D. Čakmak[1,*], Z. Tomičević[1], H. Wolf[1], Ž. Božić[1], D. Semenski[1]
[1]*University of Zagreb, Faculty of Mech. Eng. And Nav. Arch., I. Lučića 5, 10000 Zagreb, Croatia*

In this study, an analytical method for fast spectral fatigue-life prediction under Broad-Band random loading is proposed. It is based on the power spectral density of stress in the structure and the Rice peak stress distribution [1] of a stationary Gaussian random process. The novel solution is based on special HyperGeometric function [2] which analytically describes the transcendent error weighting function inherent in the Rice model. HyperGeometric-based solution possesses higher accuracy than other currently existing approximate method based on the Rice peak distribution [3]. Straightforward computational applicability of obtained results is accented. The main contribution of performed investigation is given as follows. An extensive overview for existing approximate solution for Rice distribution is presented. The utilized algorithms are given in detail. Some fundamental improvements to overviewed methods are proposed, e.g. Chaudhury and Dover and Chow and Lee approximate solutions [3]. Finally, an analytical closed-form solution to given problem is proposed by solving the Rice PDF integral via special HyperGeometric ${}_2F_1$ function. The correctness and accuracy of proposed solution is verified through extensive parametric analyses and comparison with corresponding numerical solution. Weighted sum of the normal/Gaussian and Rayleigh distributions known as Rice probability density function with respect to stress range S_r, 0th spectral moment m_0, and spectral bandwidth parameters γ and λ, may be written as

$$p_{\mathrm{Rice}}\left(S_{\mathrm{r}}\right)=\frac{1}{2\sqrt{m_0}}\left\{\overbrace{\frac{\lambda}{\sqrt{2\pi}}\mathrm{e}^{\frac{-S_{\mathrm{r}}^2}{8m_0\lambda^2}}}^{\text{Gaussian/BB}}+\overbrace{\left[1+\mathrm{erf}\left(\frac{S_{\mathrm{r}}\gamma}{\lambda\sqrt{8m_0}}\right)\right]}^{\text{weighting coefficient}}\overbrace{\frac{S_{\mathrm{r}}\gamma}{4\sqrt{m_0}}\mathrm{e}^{\frac{-S_{\mathrm{r}}^2}{8m_0}}}^{\text{Rayleigh/NB}}\right\}. \tag{1}$$

Results of integrating the Eq. (1) are given in simple closed-form as

$$I_{\mathrm{HG}}\left(S_{\mathrm{r}}\right)=\int_0^\infty S_{\mathrm{r}}^{\,m}p_{\mathrm{Rice}}\left(S_{\mathrm{r}}\right)\mathrm{d}S_{\mathrm{r}}=\frac{\left(\sqrt{8m_0}\right)^m}{2}\left[\frac{{}_2F_1\left(\frac{1}{2},\frac{3+m}{2};\frac{3}{2};\frac{-\gamma^2}{\lambda^2}\right)(1+m)\gamma^2+\lambda^{3+m}}{\lambda\sqrt{\pi}}\Gamma\left(\frac{1+m}{2}\right)+\gamma\,\Gamma\left(1+\frac{m}{2}\right)\right]. \tag{2}$$

Eq. (2) is considered to be a major contribution of this work and can be used for spectral-based fatigue calculations as-is, without additional mathematical manipulation or iterative procedures. HyperGeometric ${}_2F_1$ function is a well documented special function can be easily invoked in standard mathematically oriented software, e.g. *Mathematica* or *MATLab*.

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References

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[1,*] Corresponding author
E-mail address: damjan.cakmak@fsb.hr (D. Čakmak)